PHY1112: Assignment 12

> An extreme problem

Assigned: April 2th, 2024

Due: April 9th, 2024

Learning Objectives

1. Learn gradient descent for functions of a single variable.

Grade Breakdown

|  |  |  |
| --- | --- | --- |
| Part | 1 | Total |
| Points | 24 | 24 |
| Score |  |  |

**Question 1: Gradually to the Extreme.**

In this question, we will apply gradient descent for functions of a single variable to find the minimum of a quartic function given by:

1. Write a Python function that returns the value of of the quartic function given above for an input value of .   
   **(2 marks)**
2. Write a Python function that performs gradient descent to find the minimum of a function of one variable

The function should take as inputs:

* Function handle for
* Starting point for the gradient descent,
* Step size for numerical derivative,
* The descent rate
* The convergence condition
* The maximum number of iterations,

The output should be:

* The found minimum

The gradient descent algorithm should implement the numerical derivative as a central difference.

The code should test the following convergence condition: |

The function should print a warning message if the maximum number of iterations is reached without satisfying the convergence condition. In this case it should return nan or none.  
**(10 marks)**

1. Using your gradient descent function from part b), find the minimum of the quartic function defined above, for initial positions , ,   
   **(3 marks)**

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1. Some of your answers in part ‘c’ give different results. Why? Which answer is the global minimum? How do you know? Plot the function. Use this plot to assist with your explanation.  
   **(5 marks)**

Different results are achieved at x = 1 as opposed to x = 0 and x = -1 because there are two local minima in the graph, one being closer to x = 1, the other being closer to x = 0 and x = -1. As observed from the graph, the minima at x ≈ -1.3 is the global minima.

A graph of a function

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**Figure 1.** A graph of the function in the domain of [-2, 2].

1. What single input can you alter in order to make your gradient *descent* function find the maxima instead (that is, it performs gradient *ascent*)? Why does this happen?  
     
   Change this input, and find the local maximum between your two minima from part ‘c’  
   **(3 marks)**

By changing whether the gradient product is added or subtracted from the initial x, we can change whether the function is a gradient ascent or descent. This happens because that’s how math works. . . (am I missing something?).

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**(24 marks total, 1 for docstrings/file header/variable naming/comments)**